

# Analysis and Study Indicators for Quarter Car Model with Two Air Suspension System

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## Abstract

Modeling and simulation of non-linear quarter-car suspension system for two air spring models (traditional and dynamic new air spring) are contrasted in terms of (RMS) sprung mass acceleration, dynamic load coefficient, the vertical displacement, they are compared. Two and three (DOF) of the mathematical quarter models are implemented in MATLAB/Simulink platform. The Ride Comfort (RC), Dynamic Load Coefficient (DLC) and Road Handling (RH) responses are evaluated as objective functions respectively considering a vehicle speed at 72 km/h and road ISO Class B. The obtained results indicate that the vertical displacement, the (RMS) of the sprung mass acceleration, and dynamic load coefficient values with the new air model system decrease by 10.7 %, 30.6 %, and 13.49 % respectively, in comparison to a tradition suspension system, this one gives more comfort and effortless handling.

**Keywords:** Air suspension system, Ride comfort, Road holding, Dynamic load coefficient, and Vehicles used for commercial purposes.

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## 1. Introduction

Because of the benefits in ride comfort and control of height, air suspension systems have become more popular in recent years. The key compromise is between ride quality and driving stability, because the suspension must be soft on one side to reach a high level of comfort while being stiff enough to retain good tire contact with the ground to ensure safety. However, an air spring suspension system can be developed to improve the vertical dynamics of vehicles. Air springs provide better comfort and handling performance because simple decrease of stiffness for obtaining soft suspension and constant suspension space, independently of load between the sprung and un sprung mass moreover, by supplying and strangling the air through an air circuit connected to the air spring it is possible to adjust air springs' heights. For analyzing the effects of suspension design parameters on vehicle ride comfort as well as friendliness of the road surface when vehicles travel on the road surface of ISO class B at velocity 20 m/s with a complete load for a half-vehicle model of a semi-trailer, the RC of the vehicle with the increase of the damping of the suspension was improved by Quynh et al. [1]. The simulation findings demonstrated that independent stiffness and the ride height control tuning can enable suspension that is more adaptable design for striking a better balance between a numerous conflicting requirements and obstacles was presented by Yin et al. [2]. The active pneumatic suspension with control strategy for a commercially manufactured small car which based on mass flow control with two different control strategies were presented, the first model mass flow to the air source has to be based on the differences of velocity

between excitation and sprung mass and the mass flow rate in the second model is the sinusoidal function of the stimulation frequency from the base were presented by Razdan et al. [3].

Gavriloski et al. [4] proposed a new air dynamic spring to develop the coefficient of damping for flow resistance when two volumes are linked together for analyzing the vertical dynamics of vehicle and the simulation and experimental results were analyzed. Masliiev et al. [5] studied the effect of system components parameters on the damping factor of vibrations and the development of the vibration amplitudes in a laminar and turbulent mode of airflow by an aperture that connects the air source with the supplementary reservoir was demonstrated in a pneumatic spring suspension, and gas-thermal and dynamic phenomena. Also, showed the damping coefficient dependent on the cross-section of the throttle orifice. Quynh et al. [6] evaluated the dynamic interaction between the road and heavy vehicle when the road surface roughness is random, by using impact factor of dynamic tire load, the minimal vertical dynamic load factor, and the maximal vertical dynamic load factor. Toyofuku et al. [7] investigated the relationship between the frequency of vibration and the air spring during the effect of the pipe diameter which connect air spring with auxiliary chamber and length on the dynamic air spring. Long et al. [8] analyzed the performance (the hydro pneumatic, the rubber and air system) for a fully dynamic model of a large vehicle operates under the different road surface roughness to assess the impact of suspension features on the dynamic load coefficient (DLC) and the obtained results. The hydro-pneumatic suspension system is much superior to the other suspension systems. Tang et al. [9] showed that the amplitude and frequency of the



stimulation had an effect on the air spring's dynamic stiffness when the air spring's vertical dynamic is the only thing that gets attention, air mass and heat transfer changes in an air chamber are not taken into account, also showed also showed the experimental results were agreed with simulation results.

Buhari et al. [10] studied the simulate realistic traffic behavior, many gross-weight truck conditions were employed and the findings revealed that uneven road profiles, which excite the car as it travels, result in constantly shifting tire forces. Furthermore, the kind of suspension (steel and air) was shown to have a significant impact on dynamic loading. And showed the same load and car speed, DLCs were always less with air suspension than with steel suspension. Lee and Kim [11] designed a dual-chamber air spring designed a dual-chamber air spring with nonlinear behavior that was highly amplitude dependent and by taking two important variables into account, the complicated stiffness model was enhanced. One is to examine the diaphragm's amplitude-dependent complex stiffness, which is required for air leakage prevention. The second is to replace the unidirectional flow model with a dynamic model for oscillating flow in a capillary tube linking the two air chambers. Long et al. [12] established the performance of the suspension system's air spring in comparison to the hydro-pneumatic suspension system under different operating conditions indicated that the air suspension system's performance is superior to the hydro-pneumatic system and showed that the values ( $a_b$ ) and (DLC) of the pneumatic system decrease by 11.87 % and 41.46 % when moved on the ISO class B route at a car speed of 60 km/hr and full load. Ali and Hameed [13] proposed a model for a suspension system that replaces a coil spring with a Nishimura air spring and the damper by using a pneumatic actuator to get at the model's ultimate mathematical equalization. Quynh [14] developed a three-dimensional car-pavement coupled model with 14 degrees of freedom to analyze the influence of semi-trailer truck operating conditions on the road surface.

The objective of this study is to assess the effectiveness of the new air spring contrasted to the air suspension system by a mathematical model of both air springs and to test the performance of air suspension systems when the car is moving under a variety of conditions such as velocity and road surface roughness.

## 2. Modeling of air suspension

### 2.1. Mathematical Model for New Air Dynamic

In this work, the ride comfort (RC), road holding (RH), and dynamic load coefficient in response to the quarter model of the new air suspension system are analyzed in comparison to the traditional system under ISO according arbitrary inputs from the road. Fig.1 shows the air spring system of the physical model of the pneumatic spring comprised of an airbag linked to an auxiliary tank by a system of pipelines. The stiffness of the airbag varies according to the reservoir capacity, which is determined by the size of the airbag and the pipelines connecting the two volumes. A physical model of a simpler gas spring is used to develop a new mathematical model for a new gas spring. A mechanical barrier was introduced in the pipeline as a fictive piston to take account of the changes in gas status in both volumes. The mechanical barrier is regarded as having neglected mass, and an equivalent amount of fluid mass along the pipeline oscillating is added to the partition. The displacement of the barrier triggers modifications in pressure in both volumes. A change in gas status is

polytropic. The simplified air spring model is subject to a number of assumptions of tiny changes in gas condition and linearization that is appropriate [15].

The dynamic behavior is depicted in the equation below for the newly developed dynamic model. A force balance acting on the pneumatic spring. The structure schematic of Gensys illustrated in Fig. 2.

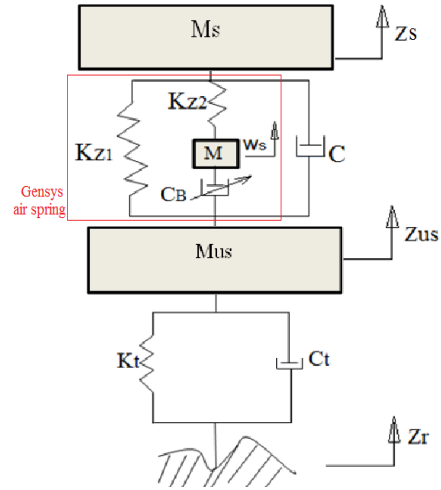


Fig. 1 Air suspension system.

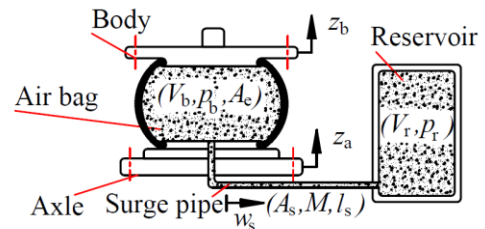


Fig. 2 Structure schematic of Gensys [16].

The vertical force of the air suspension is defined in eq. (1) [16].

$$F_z = K_{z1}(z_s - z_{us}) + K_{z2}(z_s - z_{us} - \omega_s) + C_B(\dot{z}_s - \dot{z}_{us}) \quad (1)$$

Where two linear springs that represent the spring stiffness  $K_{z1}$  and  $K_{z2}$ , make the new dynamic model (NDM) as shown in eq. (2) and (3).

$$K_{z1} = \frac{P_o n A_{eff}^2}{V_{BO} + V_{RO}} \quad (2)$$

$$K_{z2} = \frac{P_o n A_{eff}^2}{V_{BO} + V_{RO}} \frac{V_{RO}}{V_{BO}} \quad (3)$$

The bag's and reservoir's initial volumes  $V_{BO}$  and  $V_{RO}$ , and the specific heat ratio  $n$ , effective area of the air bag  $A_{eff}$ , the initial pressure in air bag  $P_o$ . The stiffness of the reservoir spring  $K_{z2}$  is related to the stiffness of air bag  $K_{z1}$  through the initial volumes by their ratio.

A nonlinear viscous damper  $C_B$  is proportional to the velocity over the damper but not to the surge pipe velocity. As a result,  $F_{vz}$  represent the vertical viscous force is denoted by the eq. (4) [17].

$$F_{vz} = K_{z2}(z - \omega_s) = C_B |\dot{\omega}_s|^\beta \text{sign}(\dot{\omega}_s) + M \ddot{\omega}_s \quad (4)$$

This equation can be rewritten as:

$$M \ddot{\omega}_s = K_{z2}(z_s - \omega_s) - C_B |\dot{\omega}_s|^\beta \text{sign}(\dot{\omega}_s) \quad (5)$$

Where the eq. (6) represent the nonlinear viscous damping  $C_B$  and eq. (7) represent the mass  $M$  of the pneumatic spring.

$$C_B = C_p \left( \frac{A_{eff} V_{RO}}{A_S V_{RO} + V_{BO}} \right)^3 \quad (6)$$

$$M = L_S A_S \rho_{air} \left( \frac{A_{eff} V_{RO}}{A_S V_{RO} + V_{BO}} \right)^2 \quad (7)$$

Where,  $C_p = 0.5 \rho_{air} A_S k_T$  and,  $k_T$  is total loss coefficient,  $L_S$  is the length of the connecting pipe,  $A_S$  is the pipeline's cross-sectional area, and  $\rho_{air}$  is the initial air density. The coefficient of total loss is defined as

$$k_T = k_{fr} + k_{en} + k_c + k_b \quad (8)$$

Where,  $k_T$  is the coefficient of total loss,  $k_{fr}$  is the frictional loss coefficient,  $k_{en}$  is the enlargement induced loss coefficient,  $k_c$  is the loss coefficient due to contraction and  $k_b$  is the pipe bend loss coefficient, respectively.

### 2.2. Mathematical Model for Air Spring (Tradition)

On the basis of the thermodynamic and fluid equations, the model of the air spring is derived. A mathematical model of passive of the quarter car of 2 DOF with air suspension system are compared under road profile with bump height 0.1 m and ISO-based random road as input. The schematic diagram of the pneumatic spring model as illustrated in Fig. 3.

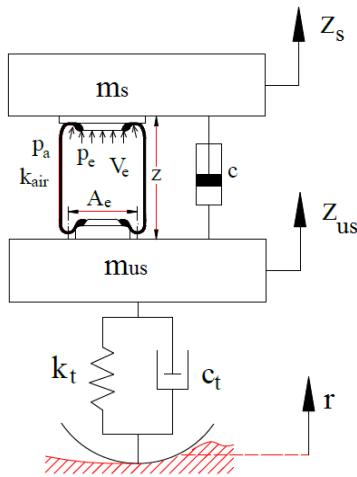


Fig. 3 tradition air spring [18].

Where,  $m_{us}$  and  $m_s$  are unsprung mass and sprung mass of car,  $k_t$  and  $c_t$  are the stiffness and damping coefficients of the tire,  $k_{air}$  and  $c$  is the stiffness coefficient of airbag and damping coefficient of a hydraulic damper,  $Z_{us}$  and  $Z_s$  are the vertical displacements of unsprung mass and sprung mass, and  $r$  is the excitation of road surface roughness. For calculating stiffness of air bag, the force balance acting on the air spring. According to [18], the eq. (9) represent the stiffness air spring.

$$k_{air} = \left[ P_a + (P_o + P_a) \left( \frac{V_o}{V_e} \right)^n - P_a \right] \frac{A_e^2}{V_e} + \frac{dA_e}{dz} \left[ (P_o + P_a) \left( \frac{V_o}{V_e} \right)^n - P_a \right] \quad (9)$$

Where,

$P_o$ : is the absolute pressure in the bag measured in (Pa).

$P_g$ : is the gauge pressure measured in (Pa).

$P_a$ : is the atmospheric pressure measured in (Pa).

$A_e$ : is the effective area ( $m^2$ ).

For isothermal procedure  $n = 1$ , for polytropic  $1 \leq n \leq \gamma$ , and for adiabatic  $n = \gamma = 1.4$  for air

The effective volume and area are referred to equations (10), (11) respectively [16].

$$V_e = V_o - \alpha(Z_s - Z_{us}) \quad (10)$$

$$A_e = A_o - \beta(Z_s - Z_{us}) \quad (11)$$

Where,  $V_e$  is the effective volume,  $V_o$  is the initial effective of volume.  $A_o$  is the initial effective area,  $A_e$  is effective area  $\alpha$  and  $\beta$  with respect to  $z$  is the change in effective volume and area.

### 3. Road profile excitation model

In this section, two types of road profiles are used to the model that have been selected by the MatLab-Simulink (MatLab, 2018). The input for irregularity of road it can take different forms as random roughness or standard humps.

1. The model is simulated using the MatLab/Simulink platform with a random road roughness as shown in Figs. 4 and 5 to be determined according to ISO/TC108 in terms of a  $Gq(n_o)$  roughness coefficient defined in Table 1 road of level B.

Where,  $Z_r(t)$  is the uneven road surface which can be obtained using eq. (12) [19].

$$\dot{Z}_r(t) = -2\pi f_o v q(t) + 2\pi n_o \sqrt{v G_q(n_o)} w(t) \quad (12)$$

Where,  $f_o = 0.0628$  Hz is a minimal boundary frequency,  $v$  is the speed in m/s which are different,  $n_o = 0.1$  as a spatial reference frequency  $m^{-1}$ ,  $w(t)$  is the Gaussian white noise,  $Gq(n_o)$  is the roughness coefficient in  $m^3$ .

Table 1. road-roughness coefficients.

Road class	Class A	Class B	Class C	Class D
$Gq(n_o) \times 10^{-6}$	16	64	256	1024

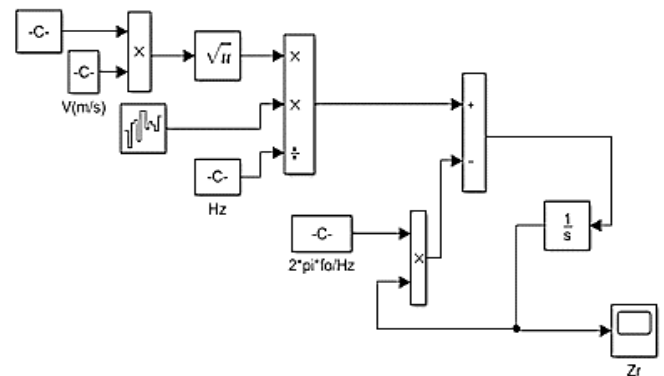


Fig. 4 Simulink road input ISO level B.

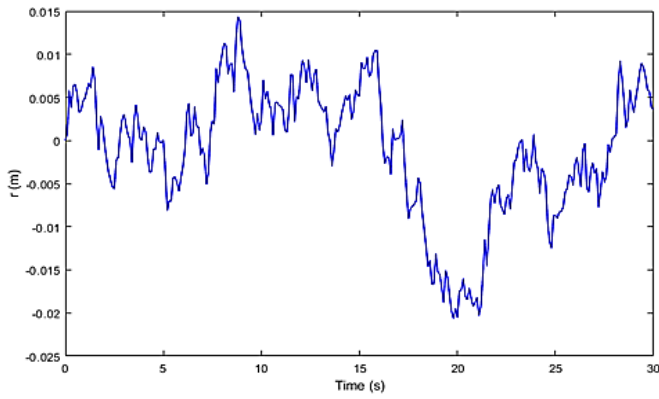


Fig. 5 Roughness road input profile ISO class B.

2. It is assumed that the second road profile as the sinusoidal road excitation derived by equation a speed hump is regarded as an input to the quarter-car model as shown in Figs. 6 and 7. Standard humps have well-known designs as circular, parabolize or flat [20]. The circular type will be considered in this analysis. The parameters of the circular speed hump as shown in eq. (13) [21].

$$y = h \sin(\omega t) \tag{13}$$

Where, the height  $h$  is 0.1 m, the length  $L$  is 5.2 m, and

$$\omega = \frac{\pi V}{3.6 L} \quad \text{rad/s}$$

The hump's equation for the time domain is as follows: longitude  $L$ , height  $h$  and speed  $V$  in km/hr.

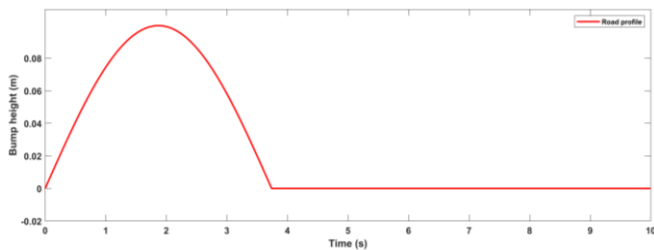


Fig. 6 Road with bump height 0.1 m.

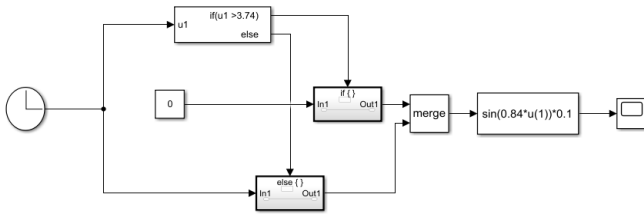


Fig. 7 Simulink for bump height 0.1 m.

## 4. Indicators of evaluation

### 4.1. Comfort of the car ride

There are numerous ways in which vehicle driving comfort can be evaluated, including method in the frequency domain, method in the time domain etc. This work is based on ISO 2631-1 in 1997. The weighted root-mean-square (RMS) acceleration was measured as part of the vibration evaluation as shown in eq. (14) [22].

$$a_w = \left[ \frac{1}{T} \int_0^T a_w^2(t) dt \right]^{1/2} \tag{14}$$

Where,  $T$  is the duration of the measurements,  $a_w$  is the weighted translational and rotational acceleration as a function of time in  $m/s^2$ . This is a way to compare value of vertical acceleration in the vehicle to the values as shown in Table 2, for evidence of likely reactions of different magnitudes of overall vibrations in public transport, to that of a synthetic index called vertical weighted rms acceleration to provide a comfortable impression of the vehicle suspension.

Table 2. Comfort levels associated with  $a_w$  values [22].

$a_w$ ( $m/s^2$ )	level of comfort
< 0.315	Not uncomfortable
0.135 - 0.63	Somewhat uncomfortable
0.5 - 1	Fully uncomfortable
0.8 - 1.6	Uncomfortable
1.25 - 2.5	So uncomfortable
> 2	Highly uncomfortable

### 4.2. Road holding

The tire's ability to stay in contact with the road is known as road holding (RH), is the distance between the road's excitation and the tire's displacement ( $Z_{u-r}$ ), and ( $Z_{u-r}$ ) is negative values indicate that the tire has lost touch with the road [23].

### 4.3. Road damage criteria

In order to analyze the impact of the various vehicle operating conditions on the comfort and handling of the vehicle, the DLC is selected as a ratio of the root-mean-square of the dynamic vertical tire force to the static force gives an objective function [6]. Evaluate the values of the weighted RMS acceleration of the vertical vehicle body according to eq. (14) and  $DLC$  according to eq. (15). Table 2 shown the parameter of comfort levels associated with  $a_w$  values.

$$DCL = \frac{F_{t,rms}}{F_s} \tag{15}$$

where,  $F_{t,rms} = K_t(Z_u - Z_r)$

Where,  $F_{t,rms}$  and  $F_s$  are the vertical dynamic in root-mean-square and the static tire force.

## 5. Simulation and analysis results

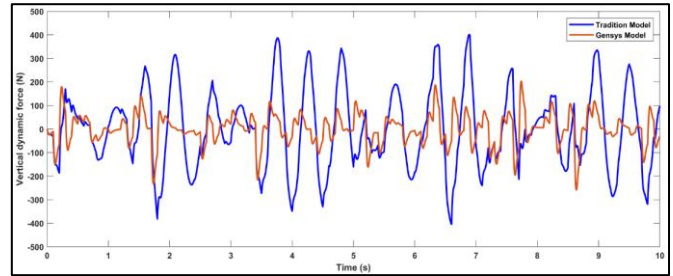
Comparisons between the new air model suspension with the traditional model are carried out to verify the new dynamic air spring suspension's efficacy. Solve the nonlinear differential equation of quarter car, equations of two models and the random road surface excitation, MatLab software/Simulink toolbox with a set of parameters are used in Table 3. The simulations are run while the vehicle is moving on sinusoidal road excitations (ISO) road surface class B.

The air spring volume and the air spring effective area constitute the most important parameters for the design of the air spring for city car [24].

Comfort of the ride and road handling of the quarter car suspension for two models are compared and the acceleration of the body's time response, vertical displacement, the dynamic load coefficient, and the dynamic tire force is assessed. A random road irregularity according to ISO/TC108 in terms of a ( $Gq$ ) roughness coefficient (level B) is used as input to the model simulations with the velocity at 72 km/hr, the time domain comparison of the new air spring and tradition suspension are shown in Figs. 8 to 12.

**Table 3.** Quarter car parameters.

Parameter	Description	Values	Unit
$m_{us}$	unsprung mass	40	kg
$m_s$	sprung mass	400	kg
$c$	damping coefficient	1500	N.s/m
$k_s$	spring stiffness	16000	N/m
$k_t$	tire stiffness	160000	N/m
$c_t$	damping coefficient of tire	750	N.s/m
$P_a$	outside ambient pressure	$1 \times 10^5$	Pa
$V_o$	initial effective volume	0.00298	$m^3$
$A_o$	initial effective area	0.01667	$m^2$
$P_o$	initial pressure in air bag	$2.682 \times 10^5$	Pa
$n$	polytropic exponent	1.4	-
$A_s$	connecting pipe cross-section	$0.0686 \times 10^{-6}$	$m^2$
$L_s$	connecting pipe length	2	m
$\rho$	density of the air	1.2	$kg/m^3$

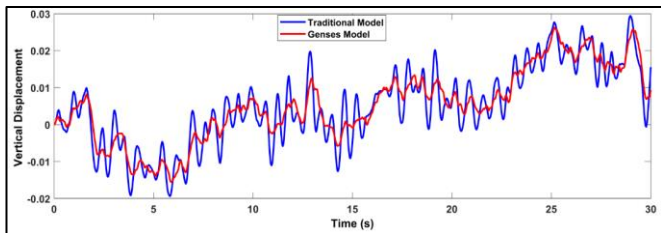


**Fig. 12** Dynamic tire load with road profile level B and velocity 72 km/hr.

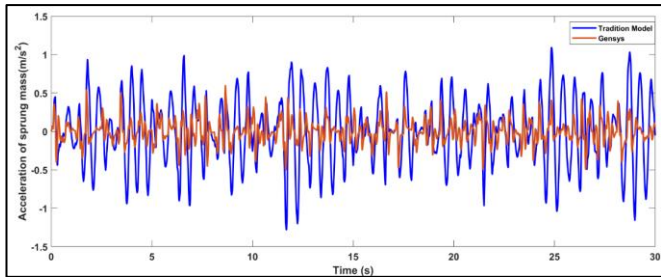
Figures 8, 9, 10, 11 and 12 indicate that the vertical displacement, ride comfort, road holding, dynamic load coefficient, dynamic tire load for the Gensys model less than the traditional model, and therefore the results show that the quarter car dynamic new air spring models are better for ride comfort and road handling compared to traditional results indicated by RMS and the response to the time domain. As explained earlier the parameters' values with the new air spring model Gensys and suspension system respectively are improved as shown in the Table 4, when compared to the traditional air spring suspension system.

**Table 4.** Reduction in RMS values for level B road input.

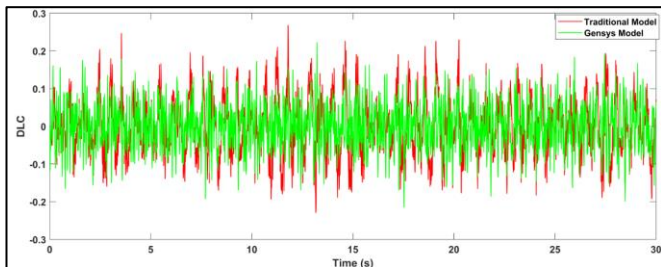
No.	Parameters	Traditional	New air dynamic (Gensys)	Reduction
1	Acceleration	4.87 e-1	3.38 e-1	30.6 %
2	Vertical displacement	8.011 e-3	7.155 e-3	10.7 %
3	Road holding	1.206 e-3	1.044 e-3	13.5 %
4	Dynamic load coefficient	6.563 e-2	5.678 e-2	13.49 %
5	Dynamic tire load	1.69 e+2	6.509 e+1	61.5 %



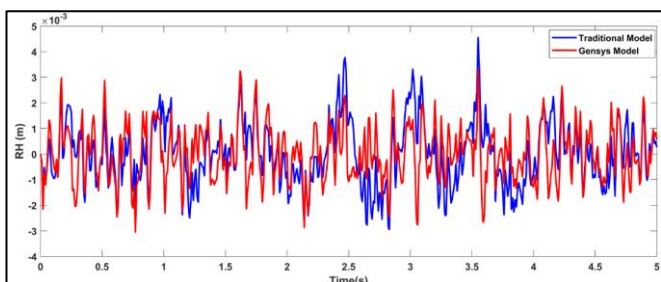
**Fig. 8** Vertical displacement of sprung body with velocity 72 km/hr with road profile level B.



**Fig. 9** Ride comfort of sprung body with velocity 72 km/hr with road profile level B.



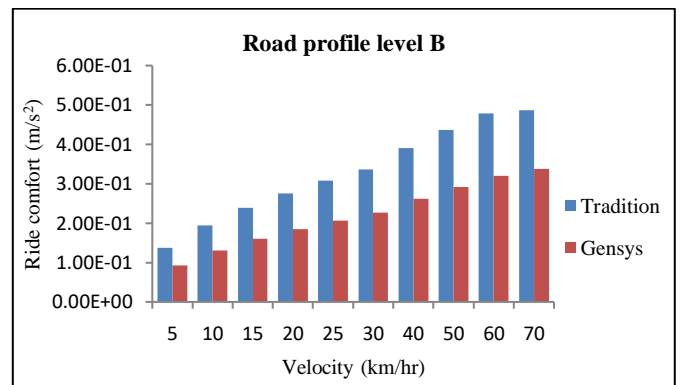
**Fig. 10** Dynamic load coefficient for velocity 72 km/hr with road profile level B.



**Fig. 11** Road holding with velocity 72 km/hr with road profile level B.

Sinusoidal road excitation is 0.1 m bump height shown in Fig. 6, and random road irregularity according to ISO/TC108 in terms of a ( $Gq$ ) roughness coefficient (level B) are used as input road profile to the models with different velocities as shown in Fig. 5.

Figures 13 to 18 showed the histogram of the effectiveness of two types of air suspension systems in mitigating the detrimental consequences of car moving over the road with bump height 0.1 m and the ISO level B road surface was considered to be comparable to the vehicle at speeds (5, 10, 15, 20, 25, 30, 40, 50, 60, and 72 m/s). The values of parameters for two input road increases with the increase of travel speed. The results show that comfort of the ride, dynamic load coefficient, and road holding of the quarter car dynamic air spring model are superior to the traditional as demonstrated by the results (RMS) and the time domain response for ride comfort, road holding, and dynamic load coefficient and this is in agreement with other studies [12] and [19].



**Fig. 13** Ride comfort with speed for road profile level B.

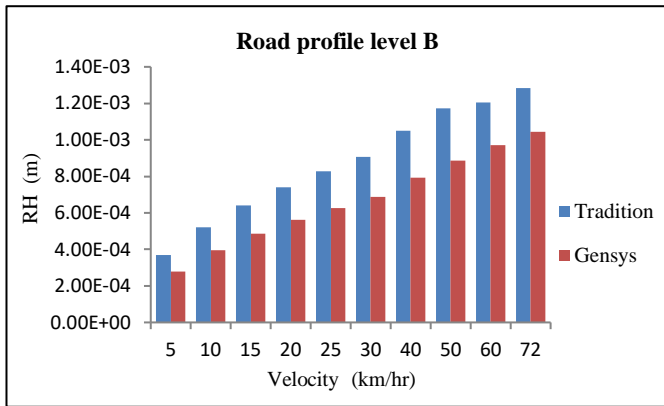


Fig. 14 Road holding with speed for road profile level B.

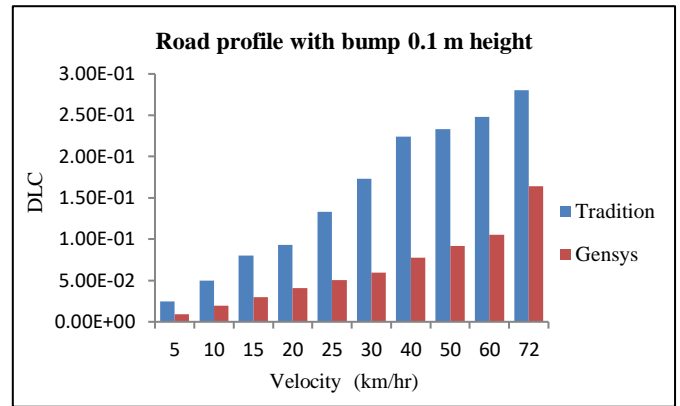


Fig. 18 Dynamic load coefficient with speed for bump height 0.1 m.

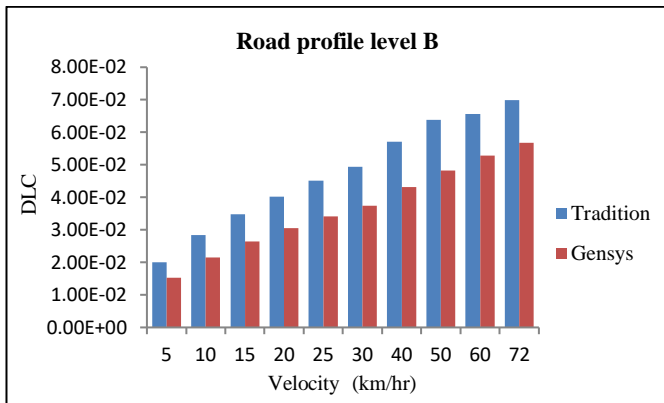


Fig. 15 Dynamic load coefficient with speed for road profile level B.

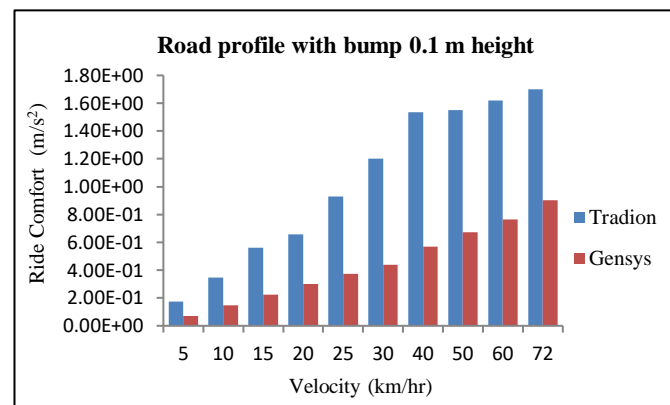


Fig. 16 Ride comfort with speed for bump height 0.1 m.

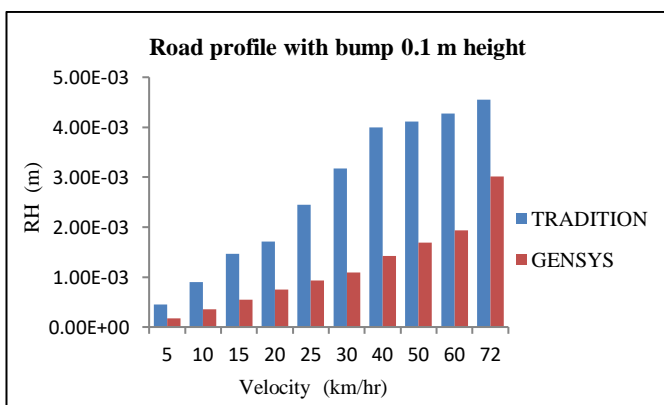


Fig. 17 Road holding with speed for bump height 0.1 m.

### 5. Conclusions

This study aims to evaluate the performance of the suspension systems for ride comfort, dynamic load coefficient and road holding by the quarter-car nonlinear model based on characteristics for the new pneumatic model and air spring suspension system.

1. The results were obtained for both uneven road bump 0.1 m moreover for the road surface of an ISO class B at ten speeds 5, 10, 15, 20, 25, 30, 40, 50, 60, and 72 km/hr with the pneumatic suspension system's performance is better than the traditional air suspension.
2. An extracted result with the different operating conditions of the vehicle's performance has been demonstrated of the new air dynamic of suspension system has a significant improvement especially, with road profile input with bump 0.1 m height for ride comfort, road holding, and dynamic load coefficient compared to the conventional suspension system.
3. A presence of the additional tank that connects with the air spring through a connecting tube, gives more damping compared to than the traditional air spring.
4. The findings will be of interest to researchers and vehicle designers for consideration during the design and testing of vehicle air suspensions.

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